Interplay between ferromagnetism and superconductivity in tunneling currents

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We study tunneling currents in a model consisting of two non-unitary ferromagnetic spin-triplet superconductors separated by a thin insulating layer. We find a novel interplay between ferromagnetism and superconductivity, manifested in the Josephson effect. This offers the possibility of tuning dissipationless currents of charge and spin in a well-defined manner by adjusting the magnetization direction on either side of the junction.

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The coexistence of ferromagnetism (FM) and superconductivity (SC) has recently been experimentally confirmed [1, 2]. This offers the possibility of observing new and interesting physical effects in transport of spin and charge. The spin-singlet character of Cooper pairs in conventional superconductors suggests that FM and SC are mutually excluding properties for a material. Indeed, coexistence of magnetic order with singlet SC is not possible for uniform order parameters [3]. On the other hand, spin-triplet Cooper pairs [4, 5] are in principle perfectly compatible with ferromagnetism. For instance, odd-in-frequency $S_z = 0$ spin-triplet superconductivity in superconductor-ferromagnet structures has been much studied in the literature [6]. The synthesis of superconductors exhibiting ferromagnetism, with simultaneously broken U(1) and O(3) symmetries, are of considerable interest from a fundamental physics point of view, and moreover opens up a vista to a plethora of novel applications. This has been the subject of theoretical research in *e.g.* Refs. [7], and has a broad range of possible applications. Also, focus on hybrid systems of ferromagnets and superconductors has arisen from the aspiration of utilizing the spin of the electron as a binary variable in device applications. This has led to spin current induced magnetization switching [8], and suggestions have been made for devices such as spin-torque transistors [9] and spin-batteries [10]. Moreover, spin supercurrents have a long tradition in ^{3}He [11], while recent work has focused on dissipationless spin-currents in unitary spin-triplet superconductors [12].

This Letter addresses the case of two p-wave superconductors arising out of a ferromagnetic metallic state, separated by a tunneling junction. Such states have been suggested to exist on experimental grounds, in compounds such as $RuSr_2GdCu_2O_8$ [13], UGe_2 [1], and URhGe [2], and have been studied theoretically in *e.g.* Refs. [14, 15, 16]. Coexisting FM and spin-triplet SC have also been proposed to arise out of half-metallic ferromagnetic materials such as CrO_2, and the alloys UNiSn and NiMnSb [17]. We compute the Josephson contribution to the tunneling currents, both in the charge- and spin-channel, within linear response theory using the Kubo formula. Our assumption is that the superconducting order is that of spin-triplet pairing, and we consider the analog of the A2-phase in ^{3}He, *i.e.* SC order parameters that satisfy $|\Delta_{\mathbf{k}\uparrow\uparrow}| \neq |\Delta_{\mathbf{k}\downarrow\downarrow}| \neq 0$ and $\Delta_{\mathbf{k}\uparrow\downarrow} = 0$. In terms of the $\mathbf{d_k}$-vector formalism [11], we then have a non-unitary state since the average spin $\langle \mathbf{S_k} \rangle = \imath \mathbf{d_k} \times \mathbf{d}^*_\mathbf{k}$ $= \frac{1}{2}(|\Delta_{\mathbf{k}\uparrow\uparrow}|^2 - |\Delta_{\mathbf{k}\downarrow\downarrow}|^2)\hat{\mathbf{z}}$ of the Cooper pairs is nonzero. Such a scenario is compatible with uniform FM and SC since the electrons responsible for ferromagnetism below the Curie temperature T_M condense into Cooper pairs with magnetic moments aligned with the magnetization below the critical temperature T_c. The choice of such a non-unitary state is motivated by the fact that there is strong reason to believe that the correct pairing symmetries in the ferromagnetic superconductors (FMSC) discovered so far are non-unitary [15, 18, 19]. The exchange field will also give rise to a Zeeman-splitting between the $\uparrow, \downarrow$ conduction bands, thus suppressing the SC order parameter $\Delta_{\mathbf{k}\uparrow\downarrow}$ [2], as illustrated in Fig. 1b).

Another important issue to address is whether the SC and FM order parameters coexist uniformly, or if they are phase-separated. One possibility is that a spontaneously formed vortex lattice due to the internal magnetization $\mathbf{m}$ is realized in a spin-triplet FMSC [20]. However, there have also been reports of uniform superconducting phases in spin-triplet FMSC [21]. A key variable determining whether a vortex lattice appears or not is the strength of the internal magnetization $\mathbf{m}$ [22]. Current experimental data on URhGe apparently do not settle this issue unambiguously, while uniform coexistence of FM and SC appear to have been experimentally verified in UGe_2 [23]. Furthermore, a bulk Meissner state in the FMSC

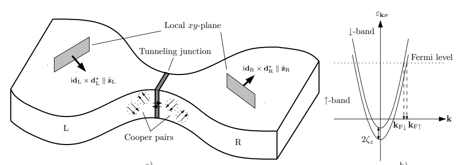

FIG. 1: a) Tunneling of Cooper pairs between two non-unitary FMSC with non-collinear magnetization. b) Band-splitting for $\uparrow, \downarrow$ electrons in the presence of a magnetization in $\hat{\mathbf{z}}$-direction, leading to a suppression of interband-pairing.

$\mathrm{RuSr_2GdCu_2O_8}$ has been reported in Ref. [24], indicating the existence of uniform FM and SC as a bulk effect. Consequently, we will use bulk values for the order parameters and assume that they coexist uniformly. We emphasize that one in general should take into account the possible suppression of the SC order parameter in the vicinity of the tunneling interface due to the formation of midgap surface states [25] which occur for certain orientations of the SC gap. The pair-breaking effect of these states in unconventional superconductors has been studied in *e.g.* Ref. [26]. A sizeable formation of such states would suppress the Josephson current, although it is nonvanishing in the general case. Also, we use bulk uniform magnetic order parameters, as in Ref. [27]. The latter is justified on the grounds that a ferromagnet with a planar order parameter is mathematically isomorphic to an s-wave superconductor, where the use of bulk values for the order parameter right up to the interface is a good approximation due to the lack of midgap surface states. Moreover, we consider thin film FMSC such that the Lorentz-force acting on the electrons will be unable to accelerate particles in a direction parallel to the junction. Our model is illustrated in Fig. 1a).

The main result of this Letter is that the Josephson current in the spin- and charge-sector between two non-unitary FMSC can be controlled by adjusting the relative magnetization orientation on each side of the junction provided that spin-triplet Cooper pairs are present. Our system consists of two FMSC separated by an insulating layer such that the total Hamiltonian can be written as $H = H_\mathrm{L} + H_\mathrm{R} + H_\mathrm{T}$, where L and R represents the individual FMSC on each side of the tunneling junction, and H_T describes tunneling of particles through the insulating layer separating the two pieces of bulk material. The FMSC Hamiltonian is given by [16] $H_\mathrm{FMSC} = H_0 + \sum_\mathbf{k} \psi_\mathbf{k}^\dagger \mathcal{A}_\mathbf{k} \psi_\mathbf{k}$, where $H_0 = JN\gamma(0)\mathbf{m}^2 + \frac{1}{2}\sum_{\mathbf{k}\sigma}\varepsilon_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\alpha\beta}\Delta_{\mathbf{k}\alpha\beta}^\dagger b_{\mathbf{k}\alpha\beta}$. Here, $\mathbf{k}$ is the electron momentum, $\varepsilon_{\mathbf{k}\sigma} = \varepsilon_\mathbf{k} - \sigma\zeta_z$, $\sigma = \uparrow,\downarrow = \pm 1$, J is a spin coupling constant, $\gamma(0)$ is the number of nearest lattice neighbors, $\mathbf{m} = \{m_x, m_y, m_z\}$ is the magnetization vector, while $\Delta_{\mathbf{k}\alpha\beta}$ is the superconducting order parameter and $b_{\mathbf{k}\alpha\beta} = \langle c_{-\mathbf{k}\beta} c_{\mathbf{k}\alpha}\rangle$ is the two-particle operator expectation value. The ferromagnetic order parameter is defined by $\zeta = 2J\gamma(0)(m_x - \imath m_y)$ and $\zeta_z = 2J\gamma(0)m_z$. We express the Hamiltonian in the basis $\psi_\mathbf{k} = (c_{\mathbf{k}\uparrow}\ c_{\mathbf{k}\downarrow}\ c_{-\mathbf{k}\uparrow}^\dagger\ c_{-\mathbf{k}\downarrow}^\dagger)^\mathrm{T}$, where $c_{\mathbf{k}\sigma}$ ($c_{\mathbf{k}\sigma}^\dagger$) are annihilation (creation) fermion operators. Note that there is no spin-orbit coupling in our model, *i.e.* inversion symmetry is not broken. Consider now the 4×4 matrix

$$\mathcal{A}_\mathbf{k} = -\frac{1}{2}\begin{pmatrix} -\varepsilon_\mathbf{k}\mathbf{1} + \boldsymbol{\sigma}\cdot\boldsymbol{\zeta} & \imath\mathbf{d}_\mathbf{k}\cdot\boldsymbol{\sigma}\sigma_y \\ (\imath\mathbf{d}_\mathbf{k}\cdot\boldsymbol{\sigma}\sigma_y)^\dagger & (\varepsilon_\mathbf{k}\mathbf{1} - \boldsymbol{\sigma}\cdot\boldsymbol{\zeta})^\mathrm{T} \end{pmatrix} \quad (1)$$

which is valid for a FMSC with arbitrary magnetization. As explained in the introduction, we will study in detail a non-unitary equal-spin pairing (ESP) FMSC as illustrated in Fig. 1a), *i.e.* $\Delta_{\mathbf{k}\uparrow\downarrow} = \Delta_{\mathbf{k}\downarrow\uparrow} = 0$, $\zeta = 0$ in Eq. (1). Since the quantization axes of the two FMSC are not aligned, one needs to include the Wigner d-function [29] denoted by $\mathcal{D}_{\sigma'\sigma}(\vartheta)$ with $j = 1/2$ to account for the fact that a $\uparrow$ spin on one side of the junction is not the same as a $\uparrow$ spin on the other side of the junction. The spin quantization axes are taken along the direction of the magnetization on each side, so that the angle ϑ is defined by $\mathbf{m}_\mathrm{R}\cdot\mathbf{m}_\mathrm{L} = m_\mathrm{R} m_\mathrm{L}\cos(\vartheta)$ where $m_i = |\mathbf{m}_i|$. The tunneling Hamiltonian then reads $H_\mathrm{T} = \sum_{\mathbf{k}\mathbf{p}\sigma\sigma'} \mathcal{D}_{\sigma'\sigma}(\vartheta)(T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\sigma}^\dagger d_{\mathbf{p}\sigma'} + T_{\mathbf{k}\mathbf{p}}^* d_{\mathbf{p}\sigma'}^\dagger c_{\mathbf{k}\sigma})$, where we neglect the possibility of spin-flips in the tunneling process. The validity of the tunneling Hamiltonian approach requires that the applied voltage across the junction is small. Here, we will be concerned with the case of zero bias voltage, so that the tunneling Hamiltonian approach is appropriate. Note that we distinguish between fermion operators on each side of the junction corresponding to $c_{\mathbf{k}\sigma}$ and $d_{\mathbf{k}\sigma}$. Furthermore, we write the superconducting order parameters as $\Delta_{\mathbf{k}\sigma\sigma} = |\Delta_{\mathbf{k}\sigma\sigma}|\mathrm{e}^{\imath(\theta_\mathbf{k} + \theta_{\sigma\sigma}^\mathrm{R})}$, where R (L) denotes the bulk superconducting phase on the right (left) side of the junction while $\theta_\mathbf{k}$ is an internal phase factor originating from the specific form of the gap in $\mathbf{k}$-space that ensures odd symmetry under inversion of momentum, *i.e.* $\theta_\mathbf{k} = \theta_{-\mathbf{k}} + \pi$.

For our system, the Hamiltonian takes the form $H_\mathrm{FMSC} = H_0 + H_A$, $H_A = \sum_{\mathbf{k}\sigma}\phi_{\mathbf{k}\sigma}^\dagger A_{\mathbf{k}\sigma}\phi_{\mathbf{k}\sigma}$, where we have chosen a convenient basis $\phi_{\mathbf{k}\sigma}^\dagger = (c_{\mathbf{k}\sigma}^\dagger, c_{-\mathbf{k}\sigma})$ that block-diagonalizes $\mathcal{A}_\mathbf{k}$, and defined $A_{\mathbf{k}\sigma} = \frac{1}{4}[2\varepsilon_{\mathbf{k}\sigma}\sigma_z + \Delta_{\mathbf{k}\sigma\sigma}(\sigma_x + \imath\sigma_y) + \Delta_{\mathbf{k}\sigma\sigma}^\dagger(\sigma_x - \imath\sigma_y)]$ with Pauli matrices σ_i. This Hamiltonian is diagonalized by a 2×2 spin generalized unitary matrix $U_{\mathbf{k}\sigma}$, so that the superconducting sector is expressed in the diagonal basis $\tilde{\phi}_{\mathbf{k}\sigma}^\dagger = \phi_{\mathbf{k}\sigma}^\dagger U_{\mathbf{k}\sigma} \equiv (\gamma_{\mathbf{k}\sigma}^\dagger, \gamma_{-\mathbf{k}\sigma})$. Thus, $H_A = \sum_{\mathbf{k}\sigma}\tilde{\phi}_{\mathbf{k}\sigma}^\dagger \tilde{A}_{\mathbf{k}\sigma}\tilde{\phi}_{\mathbf{k}\sigma}$, in which $\tilde{A}_{\mathbf{k}\sigma} = U_{\mathbf{k}\sigma}A_{\mathbf{k}\sigma}U_{\mathbf{k}\sigma}^{-1} = \mathrm{diag}(\widetilde{E}_{\mathbf{k}\sigma}, -\widetilde{E}_{\mathbf{k}\sigma})/2$, and $\widetilde{E}_{\mathbf{k}\sigma} = \sqrt{\varepsilon_{\mathbf{k}\sigma}^2 + |\Delta_{\mathbf{k}\sigma\sigma}|^2}$.

In order to find the spin and charge currents over the junction, consider first the generalized number operator $N_{\alpha\beta} = \sum_\mathbf{k} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\beta}$. The transport operator in the interaction picture then reads $\dot{N}_{\alpha\beta}(t) = -\mathrm{i}\sum_{\mathbf{k}\mathbf{p}\sigma}[\mathcal{D}_{\sigma\beta}(\vartheta)T_{\mathbf{k}\mathbf{p}}c_{\mathbf{k}\alpha}^\dagger(t)d_{\mathbf{p}\sigma}(t)\mathrm{e}^{-\mathrm{i}teV} - \mathcal{D}_{\sigma\alpha}(\vartheta)T_{\mathbf{k}\mathbf{p}}^* d_{\mathbf{p}\sigma}^\dagger(t)c_{\mathbf{k}\beta}(t)\mathrm{e}^{\mathrm{i}teV}]$, where $eV \equiv \mu_\mathrm{L} - \mu_\mathrm{R}$ is an externally applied potential. The general current across the junction can be written

$$\mathbf{I}(t) = \sum_{\alpha\beta}\boldsymbol{\tau}_{\alpha\beta}\langle\dot{N}_{\alpha\beta}(t)\rangle,\ \boldsymbol{\tau} = (-e\mathbf{1}, \boldsymbol{\sigma}) \quad (2)$$

such that the charge-current is $I^\mathrm{C}(t) = I_0(t)$ while the spin-current reads $\mathbf{I}^\mathrm{S}(t) = (I_1(t), I_2(t), I_3(t))$. Note that Eq. (2) contains both the single-particle (sp) and two-particle (tp) contribution. The concept of a spin-current in this context refers to the rate at which the spin-vector $\mathbf{S}$ on one side of the junction changes *as a result of tunneling across the junction*, *i.e.* $\dot{\mathbf{S}} = \imath[H_\mathrm{T}, \mathbf{S}]$. As there is no spin-orbit coupling in our system, this definition

of the spin-current serves well [30]. The spatial components of $\mathbf{I}^{\mathrm{S}}$ are defined with respect to the corresponding quantization axis. We compute the currents by the Kubo formula, $\langle \dot{N}_{\alpha\beta}(t)\rangle = -\mathrm{i}\int_{-\infty}^{t} \mathrm{d}t' \langle [\dot{N}_{\alpha\beta}(t), H_{\mathrm{T}}(t')]\rangle$, where the right hand side is the statistical expectation value in the unperturbed quantum state, *i.e.* when the two subsystems are not coupled. We now focus on the two-particle charge-current and $\hat{\mathbf{z}}$-component of the spin-current, such that only $\alpha = \beta$ contributes in Eq. (2).

Using linear response theory with the Matsubara formalism, one arrives at $\langle \dot{N}_{\alpha\alpha}(t)\rangle_{\mathrm{tp}} = 2\sum_\sigma \Im\mathrm{m}\{\Psi_{\alpha\sigma}(eV)\mathrm{e}^{-2\mathrm{i}teV}\}$, where $\Psi_{\alpha\sigma}(eV)$ is obtained by performing analytical continuation $\imath\omega_n \to eV + \imath 0^+$ ($\omega_n = 2\pi n k_{\mathrm{B}}T, n = 1,2,3\ldots$) on $\widetilde{\Psi}_{\alpha\sigma}(\imath\omega_n) = -\int_0^{1/k_{\mathrm{B}}T} \mathrm{d}\tau \mathrm{e}^{\imath\omega_n\tau}\langle \mathrm{T}_\tau M_{\alpha\sigma}(\tau)M_{\alpha\sigma}(0)\rangle$, where $M_{\alpha\sigma}(t) = \sum_{\mathbf{kp}} \mathcal{D}_{\sigma\alpha}(\vartheta) T_{\mathbf{kp}} c^\dagger_{\mathbf{k}\alpha}(t) d_{\mathbf{p}\sigma}(t)$, while T is the temperature, and T_τ denotes the time-ordering operator. Explicitly, we find

$$\Psi_{\alpha\sigma}(eV) = -\sum_{\substack{\mathbf{kp}\\ \lambda,\rho=\pm}} \mathcal{D}^2_{\sigma\alpha}(\vartheta)|T_{\mathbf{kp}}|^2 \frac{\Delta^*_{\mathbf{k}\alpha\alpha}\Delta_{\mathbf{p}\sigma\sigma}}{4E_{\mathbf{k}\alpha}E_{\mathbf{p}\sigma}} \Lambda^{\lambda\rho}_{\mathbf{kp}\alpha\sigma}(eV) \tag{3}$$

where $E_{\mathbf{k}\sigma} = \sqrt{\xi^2_{\mathbf{k}\sigma} + |\Delta_{\mathbf{k}\sigma\sigma}|^2}$, $\xi_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}\sigma} - \mu_{\mathrm{R}}$ ($\mathrm{R} \to \mathrm{L}$ for $\mathbf{k} \to \mathbf{p}$), and $\Lambda^{\lambda\rho}_{\mathbf{kp}\alpha\sigma}(eV) = \lim_{\imath\omega_n \to eV+\imath 0^+} \lambda[f(E_{\mathbf{k}\alpha}) - f(\lambda\rho E_{\mathbf{p}\sigma})]/[\mathrm{i}\omega_\nu + \rho E_{\mathbf{k}\alpha} - \lambda E_{\mathbf{p}\sigma}]$; $\lambda, \rho = \pm 1$.

In Eq. (3), we have used that $T_{-\mathbf{k},-\mathbf{p}} = T^*_{\mathbf{kp}}$ which follows from time reversal symmetry, and $f(x)$ is the Fermi distribution. Note that the chemical potential has been included in the excitation energies $E_{\mathbf{k}\alpha}$. In general, Eq. (3) will give rise to a term proportional to $\cos(\theta^{\mathrm{L}}_{\sigma\sigma} - \theta^{\mathrm{R}}_{\alpha\alpha})$, the quasiparticle interference term, in addition to $\sin(\theta^{\mathrm{L}}_{\sigma\sigma} - \theta^{\mathrm{R}}_{\alpha\alpha})$, identified as the Josephson current. In the following, we shall focus on the latter while a comprehensive treatment of all terms will be given in Ref. [31]. Consider now the case of zero externally applied voltage ($eV = 0$). From Eq. (2), we see that the Josephson charge-current becomes

$$\begin{aligned} I^{\mathrm{C}}_{\mathrm{J}} = e &\sum_{\mathbf{kp}\sigma\alpha} [1 + \sigma\alpha\cos\vartheta]\cos(\theta_{\mathbf{p}} - \theta_{\mathbf{k}})\sin(\theta^{\mathrm{L}}_{\sigma\sigma} - \theta^{\mathrm{R}}_{\alpha\alpha}) \\ &\times |T_{\mathbf{kp}}|^2 |\Delta_{\mathbf{k}\alpha\alpha}||\Delta_{\mathbf{p}\sigma\sigma}| F^{\alpha\sigma}_{\mathbf{kp}}/(E_{\mathbf{k}\alpha}E_{\mathbf{p}\sigma}), \end{aligned} \tag{4}$$

with $F^{\alpha\sigma}_{\mathbf{kp}} = \sum_\pm [f(\pm E_{\mathbf{k}\alpha}) - f(E_{\mathbf{p}\sigma})]/(E_{\mathbf{k}\alpha} \mp E_{\mathbf{p}\sigma})$ while the expression for $I^{\mathrm{S}}_{\mathrm{J},z}$ is equal except for a factor $(-\alpha/e)$ inside the summation. Observing that Eq. (4) may be cast into the form $I_{\mathrm{J}} = I_0 + I_m\cos(\vartheta)$, we have thus found a Josephson current, for both spin and charge, that can be tuned in a well-defined manner by adjusting the relative orientation ϑ of the magnetization vectors (For corresponding results in spin-singlet superconductors with helimagnetic order, see Refs. [28, 32]). Below, we discuss the detection of such an effect.

In the limit where one of the superconducting order parameters vanishes internally on both sides, *i.e.* the equivalent of an A1-phase, we see that the interplay between ϑ and $\theta_{\sigma\sigma}$ remains, as only one term contributes to the spin sum over $\{\sigma, \alpha\}$. In this case, the charge- and spin-current goes as $\cos^2(\vartheta/2)\sin\Delta\theta_{\sigma\sigma}$, where $\Delta\theta_{\sigma\sigma} \equiv \theta^{\mathrm{L}}_{\sigma\sigma} - \theta^{\mathrm{R}}_{\sigma\sigma}$ and $\Delta_{\mathbf{q}\sigma\sigma}$ with $\sigma = \{\uparrow, \downarrow\}$, $\mathbf{q} = \{\mathbf{k}, \mathbf{p}\}$ is the surviving order parameter. For collinear magnetization ($\vartheta = 0$), an ordinary Josephson effect driven by the superconducting phase occurs. Interestingly, one is able to tune this current to zero for $\mathbf{m}_{\mathrm{L}} \parallel -\mathbf{m}_{\mathrm{R}}$ ($\vartheta = \pi$).

Another result that can be extracted from Eq. (4) is a persistent spin-Josephson current even if the magnetizations on each side of the junction are of equal magnitude and collinear ($\vartheta = 0$). This is quite different from the Josephson-like spin-current recently considered in ferromagnetic metal junctions [27]. There, a twist in the magnetization across the junction is required to drive the spin-Josephson effect. In this Letter, however, we have found a persistent spin-current in the two-particle channel even for collinear magnetization.

In the special case of $eV = 0$ and equal SC phases on each side of the junction, *i.e.* $\theta^{\mathrm{L}}_{\sigma\sigma} = \theta^{\mathrm{R}}_{\sigma\sigma}$, Eq. (4) reduces to the form $I^{\mathrm{S}}_{\mathrm{J},z} = J_0\sin^2(\vartheta/2)\sin(\theta^{\mathrm{L}}_{\downarrow\downarrow} - \theta^{\mathrm{R}}_{\uparrow\uparrow})$ while $I^{\mathrm{C}}_{\mathrm{J}} = 0$. This means that a *two-particle spin-current without any charge-current* can arise for non-collinear magnetizations on each side of the junction in the absence of an externally applied voltage *and* with equal SC phases $\theta^{\mathrm{L}}_{\sigma\sigma} = \theta^{\mathrm{R}}_{\sigma\sigma}$; see, however, Ref. [33].

It is well-known that for tunneling currents flowing in the presence of a magnetic field that is perpendicular to the tunneling direction, the resulting flux threading the junction leads to a Fraunhofer-like variation in the DC Josephson effect, given by a multiplicative factor $\sin(\pi\Phi/\Phi_0)/(\pi\Phi/\Phi_0)$ in the critical current. Here, $\Phi_0 = \pi\hbar/e$ is the elementary flux quantum, and Φ is the total flux threading the junction due to a magnetic field. However, this is not an issue in the present case since the magnetization is assumed to be oriented according to Fig. 1a). Since the motion of the Cooper-pairs is also restricted by the thin-film structure, there is no orbital effect from such a magnetization.

Note that the interplay between ferromagnetism and superconductivity is manifest in the charge- as well as spin-currents, the former being readily measurable. Since the critical Josephson currents depend on the relative magnetization orientation, one is able to tune these currents in a well-defined manner by varying ϑ. This can be done by applying an external magnetic field in the plane of the FMSC. In the presence of a rotating magnetic moment on either side of the junction, the Josephson currents will thus vary according to Eq. (4). Depending on the relative magnitudes of I_0 and I_m, the sign of the critical current may change. Note that such a variation of the magnetization vectors must take place in an adiabatic manner so that the systems can be considered to be in, or near, equilibrium at all times. Our predictions can thus be verified by measuring the critical

current at $eV = 0$ for different angles ϑ and compare the results with our theory. Recently, it has been reported that a spin-triplet supercurrent, induced by Josephson tunneling between two s-wave superconductors across a ferromagnetic metallic contact, can be controlled by varying the magnetization of the ferromagnetic contact [34]. Moreover, detection of induced spin-currents are challenging, although recent studies suggest feasible methods of measuring such quantities [35]. Observation of macroscopic spin-currents in superconductors may also be possible via angle resolved photo-emission experiments with circularly polarized photons [36], or in spin-resolved neutron scattering experiments [37].

We briefly mention our results in the single-particle channel where we find that the charge-current and the $\hat{\mathbf{z}}$-component of the spin-current both vanish for $eV = 0$; see Ref. [31] for details. They are nonzero for $eV \neq 0$ even if the magnetization vectors are collinear. We stress that the finding of a non-persistent $\hat{\mathbf{z}}$-component of the spin-current does not conflict with the results of Ref. [27], as their $\hat{\mathbf{z}}$-direction corresponds to a vector in the xy-plane in our system. For $\Delta_{\mathbf{k}\sigma\sigma} \to 0$, $\mathbf{I}^{\mathrm{S}}_{\mathrm{sp}}(t) = 2\sum_{\mathbf{kp}}\sum_{\alpha\beta\sigma}\mathcal{D}_{\sigma\alpha}(\vartheta)\mathcal{D}_{\sigma\beta}(\vartheta)|T_{\mathbf{kp}}|^2\Im\mathrm{m}\{\boldsymbol{\sigma}_{\beta\alpha}\Lambda^{1,1}_{\beta\sigma}(-eV)\}$, and the component of the spin-current parallel to $\mathbf{m}_{\mathrm{L}} \times \mathbf{m}_{\mathrm{R}}$ is seen to vanish for $\vartheta = \{0, \pi\}$ at $eV = 0$ in agreement with Ref. [27].

We reemphasize that the above ideas should be experimentally realizable by *e.g.* utilizing various geometries in order to vary the demagnetization fields. One may also use exchange biasing to an anti-ferromagnet to achieve non-collinearity [38]. We have found an interplay between FM and SC in the Josephson channel for charge- and spin-currents when considering non-unitary spin-triplet ESP FMSC with coexisting and uniform ferromagnetic and superconducting order.

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[29] E. P. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren* Frieder. Vieweg, Braunschweig, 1931). $\mathcal{D}_{\uparrow\uparrow}(\vartheta) = \mathcal{D}_{\downarrow\downarrow}(\vartheta) = \cos(\vartheta/2), \mathcal{D}_{\uparrow\downarrow}(\vartheta) = -\mathcal{D}_{\downarrow\uparrow}(\vartheta) = -\sin(\vartheta/2)$; a spin-rotated fermion operator reads $\tilde{d}_{\mathbf{p}\sigma} = \sum_{\sigma'}\mathcal{D}_{\sigma'\sigma}(\vartheta)d_{\mathbf{p}\sigma'}$.
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[33] This effect requires that $\theta^{\mathrm{L(R)}}_{\uparrow\uparrow} - \theta^{\mathrm{L(R)}}_{\downarrow\downarrow} \neq \{0, \pi\}$. If an interband Josephson coupling is present, the phases will be locked according to $\theta_{\uparrow\uparrow} - \theta_{\downarrow\downarrow} = \{0, \pi\}$. It has recently [J. Shi and Q. Niu, cond-mat/0601531 (2006)] been proposed that p-wave SC could be induced by means of spin-orbit coupling in FM metals, which in turn leads to an interband Josephson coupling. However, in the general case Eq. (4) is non-zero even if the spin-up and spin-down phases are locked to each other.
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